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Solution of Nonhomogeneous Helmholtz Equation with Variable Coefficient Using Boundary Domain Integral Method

R. F. Mardanov[1*] and S. K. Zaripov[1]**

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[1] *Kazan (Volga Region) Federal University, ul. Kremlevskaya 18, Kazan, Tatarstan, 420008 Russia*

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Abstract—The Boundary Domain Integral Method (BDIM) is applied to the solution of the nonhomogeneous Helmholtz equation with variable coefficient. The analytical formulas for the integrals over the individual boundaries and domain integrals are used to increase the accuracy of the numerical approach. Comparisons of the developed BDIM with the analytical solutions for the homogeneous Helmholtz equation with constant coefficient and the nonhomogeneous Helmholtz equation with variable coefficient are given.

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1. INTRODUCTION

In order to solve the partial differential equations (PDE) various numerical methods are applied. The finite element method (FEM), finite difference method, finite volume method, boundary element method (BEM) and various meshless methods are applied to the solution of boundary value problems for PDE. The main advantage of BEM and meshless methods is that they are a relatively simple way to find the PDE solution in complex irregular domains. The additional advantage of the BEM is the reduction of the dimensionality of the problem by one unit [1–3]. A wide review of the developing BEM's and known BEM variants such as collocation BEM, galerkin BEM, dual reciprocity BEM, complex variable BEM and analog equation methods is given by [4]. Traditional BEM is used for the solution of linear, homogeneous and elliptic differential equations with constant coefficients. In the case of the nonhomogeneous equations with variable coefficients the additional extensions of the method are applied.

In [5] an finite element—boundary element coupling method has been developed to analyze the nonlinear elastoplastic problems of solid mechanics. The coupling of the two methods uses the division of the whole domain into two parts in which the methods are applied separately and the interface between the two sub-regions is performed through an iterative procedure. Reference [6] considers two-dimensional time harmonic fluid-structure interaction problems when the fluid is at rest, and the elastic bodies have small thicknesses. A BEM-FEM numerical approach is used, where the BEM is applied to the fluid, and the structural FEM is applied to the thinelastic bodies. Various combinations of the coupling of BEM and FEM are also developed in [7].

One the widely used PDE in various branches of mathematical physics and engineering equation is the Helmholtz equation. The performance of the boundary and finite element methods for the Helmholtz equation in two dimensions was investigated by [8]. The numerical investigations show that the BEM is generally more accurate than the FEM when the size of the finite elements is comparable to that of the boundary elements. Exceptions occur in the neighborhood of isolated points of the Helmholtz constant

[*] E-mail: Renat.Mardanov@kpfu.ru

[**] E-mail: Shamil.Zaripov@kpfu.ru